Financial Engineering

Unit 1 Derivatives Contracts

Contents

Unit Overview

Unit 1 will introduce some fundamental terminology relating to derivatives, and will describe some basic features of derivatives contracts. You will examine the characteristics of forward contracts, futures contracts and options, and you will learn about the types of traders who deal in derivatives, and some of the dangers of their misuse. You will also use data tables to construct profit patterns for options.

Learning outcomes

When you have completed your study of this unit and its readings, you will be able to:

- discuss and differentiate between the most common types of financial derivatives: forward contracts, futures and options
- explain the advantages of long and short positions in these contracts
- distinguish between put and call options
- outline three main reasons for the use of derivatives
- discuss the potential dangers in misusing derivatives, and how to avoid them
- conduct sensitivity analysis using data tables.

Reading for Unit 1

John C Hull (2018) *Options, Futures, and Other Derivatives.* Essex UK, Pearson. Chapter 1 ‘Introduction’, Chapter 2 ‘Mechanics of futures markets’ and Chapter 36 ‘Derivatives mishaps and what we can learn from them’.

Simon Benninga (2014) *Financial Modeling*. Cambridge MA, The MIT Press. Chapter 31 ‘Data tables’.

1.1 Introduction

Derivatives are a general class of financial contracts, which are written in terms of an underlying financial or real asset. Their payoff over a given period of time will depend on the performance of the underlying asset, which could include stocks, interest rates or exchange rates. The value of a financial derivative, therefore, depends on the performance of the underlying asset. Financial derivatives include *futures* and *options*. They are a flexible form of financial instrument, and can be very effective in enabling investors to achieve a complex risk profile. In particular, derivatives can be a very powerful instrument for reducing risk, or *hedging*. However, derivatives can also be used to increase the risk of investors, if they are used for *speculation*. Finally, derivatives allow investors to make riskless profits by exploiting mispricing of assets when they are used for *arbitrage*.

The application section in this unit provides an introduction to data tables. Data tables are used widely in financial modelling for sensitivity analysis. In this section you will construct the profit pattern for a call option on a stock. In later units you will study how to use data tables to analyse more sophisticated strategies.

We begin by looking at types of derivative contracts.

1.2 Forward Contracts

The simplest derivative instrument is a *forward contract*. This is 'an agreement to buy or sell an asset at a certain future time for a certain price' (Hull, 2018: p. 28). You can see that the main feature of forward contracts is that it enables you to *fix now* the price at which a transaction will take place *at a future date*. You can therefore make use of a forward contract in order to eliminate the uncertainty associated with the future price of an asset. A forward contract can be contrasted to a *spot contract*, where delivery of the asset is immediate. Thus, a forward contract can effectively eliminate the uncertainty of the future spot price.

The reduction of uncertainty could be desirable from the point of view of risk management. However, it is important you realise that the elimination of uncertainty does not imply that you would always be better off by entering a forward contract. Indeed, if you successfully hedge your position you will be able to eliminate or reduce the losses associated with adverse movements in the price of the underlying asset, but at the cost of foregoing the profits associated with potentially favourable asset price changes.

In order to see this point, denote the delivery price written in the forward contract by K and the future spot price on the delivery date by S_T. If you have a *long position* in the forward contract – that is, if you commit yourself to purchasing the underlying asset at contract maturity – your payoff upon maturity will be

$$S_T - K \tag{1.1}$$

The value K is written into the contract, but S_T is the spot price that will prevail in the market at the future delivery date, and cannot be known in advance. If S_T turns out to be greater than K, you will be able to purchase at a price K an asset that has the greater market price S_T, and will therefore make a profit: $S_T - K > 0$. However, if the market price S_T happens to be less than K, you will have to pay the price K for an asset whose market price is only S_T, and will therefore make a loss: $S_T - K < 0$.

If you commit yourself to *selling* an asset, you will have a *short position* in the forward contract. In this case, your payoff at maturity will be given by:

$$K - S_T \tag{1.2}$$

Your profits and losses will now be the mirror image to those in a long position. You will make a gain if the market price S_T is less than the forward price K, and will make a loss if S_T is greater than K on maturity.

If you have a long position in an asset, the gains or losses from a long position in a forward contract written on that asset will offset the losses or gains from your position in the underlying asset. You will therefore be able to use forward contracts in order to achieve a hedged position, so that your payoff is no longer subject to the asset price uncertainty.

Reading 1.1

Please now stop and read the introduction to Chapter 1 to the end of the section on forward contracts of Hull (2018), on pages 23–30. Pay special attention to the numerical examples that motivate the use of forward contracts.

Hull (2018) Chapter 1 'Introduction', sections 1.1–1.3 in *Options, Futures, and Other Derivatives*.

Exercise 1.1

Please now solve problem 1.5 on page 41 of Hull (2018). An answer is provided.

1.3 Futures Contracts

The forward contracts that you studied in Section 1.2 are usually arranged *over-the-counter* (OTC) by two financial institutions, or by a financial institution and one of its clients (see the discussion in Section 1.2 on pages 25–26 of Hull (2018)). This implies that the characteristics of the contract can be customised to the specific needs of the two parties. However, over-the-counter contracts present the potential disadvantage of involving some credit risk (that is, the risk that the counterparty may default on its obligations), and could also suffer from limited liquidity. For these reasons, clearing houses are increasingly required for some OTC transactions.

By contrast, *futures contracts* have similar features to forward contracts, but are normally traded on an *exchange market*. This requires that futures contracts be standardised – that is, they must specify in detail the characteristics of the particular underlying asset, and must follow standard conventions regarding the contract size, the delivery arrangements and the delivery dates. This both significantly reduces the credit risk associated with futures contracts and makes them much more liquid.

Futures contracts are traded on markets such as the CME Group (created in 2007 from the merger between the Chicago Board of Trade (CBOT) and the Chicago Mercantile Exchange (CME)), where trade instructions are carried out by brokers. As a *bona fide* insurance against the risk of default, brokers usually require that the investor deposits a fund known as the *initial margin*. The initial margin can be modified over the life of the contract, depending on the changes in the market value of the futures contract (*variation margin*). In turn, brokers must maintain margins with a *clearing house*, which acts as an intermediary in futures transactions. These are the *clearing margins*. The main role of margins is to reduce the credit risk in futures contracts.

Reading 1.2

Please now read Section 1.4 on page 30 and Chapter 2, pages 46–66 of Hull (2018). Section 1.4 provides a brief introduction to futures contracts, whereas Chapter 2 describes the mechanics of futures margins, including a detailed discussion of the workings of margins. Table 2.3 on page 65 usefully summarises the main differences between forward and futures contracts.

Hull (2018) Section 1.4 'Futures contracts', & Chapter 2 'Mechanics of futures markets' in *Options, Futures, and Other Derivatives.*

Exercise 1.2

Please solve problem 1.6 on page 41 of Hull (2018).

1.4 Options

Forward and futures contracts constitute a binding commitment for investors. If an investor has a long position in a forward contract, for instance, she is committing herself to completing the contract at maturity and to purchasing the underlying asset. Forward and futures contracts therefore require final delivery, unless they are closed out prior to maturity (which is typically the case for futures contracts). By contrast, *options contracts* confer onto their holders the right to buy or sell an asset at or before maturity, but do not prescribe that the purchase or sale must be executed. In other words, there is no obligation involved to carry out the transaction. If it is not profitable to do so, the holder of an option contract could choose to let the contract expire unexercised. Their only loss would be the initial cost of the option contract.

An option to buy an asset is a *call option*, and an option to sell an asset is a *put option*. The price at which the purchase or the sale can be carried out is called the *strike price*, or the *exercise price*. The price of the option contract is called the *option premium*. Option contracts that can be exercised any time up to and including the expiration date are called *American options*, whereas contracts that can only be exercised on the expiration date are called *European options*.

Investors who have the right to buy or to sell an asset (in call options or in put options, respectively) are said to have a *long* position. Their counterparties, who have issued the options, are said to have a *short* position: the options would be exercised against them.

The main feature of options contracts is that their payoffs can be strongly asymmetric. For instance, if you have a long position in a call option, the most you can lose is the premium that you paid for the option. By contrast, your gains could potentially be very large, since if the price of the asset increases on maturity you could profit from the whole difference between the spot price of the asset and the exercise price. The asymmetry in the payoff profiles of options makes them a very versatile financial instrument.

In this module you will examine in detail a number of different option contracts, and you will see how they can be combined to achieve a large variety of risk profiles for your financial portfolios.

Reading 1.3

Please now stop and read Section 1.5 of Hull (2018), pages 30–33. Pay particular attention to the two diagrams in Figure 1.3 on page 33, which illustrate the payoff profiles associated with a long position in a call option and a short position in a put option on a stock.

Hull (2018) Chapter 1, section 1.5 'Options' in *Options, Futures, and Other Derivatives.*

Exercise 1.3

Please solve problem 1.7 on page 42 of the book by Hull.

1.5 Types of Traders

As noted already, derivatives are very flexible financial instruments. They can be used for a variety of purposes, and can be very effective in modifying the risk profile of investors.

There are three main reasons for the use of derivatives.

- First, they can be used to reduce risk – that is, for *hedging*. For instance, if an investor has a long position in an asset she can hedge her risk by taking a short position on a derivative written on the underlying asset.
- Second, derivatives can be used in order to increase risk – that is, for *speculation*. If you believe that the market is mispricing the probability that the price of an asset will change – for instance, you could believe that it is more likely that the price of an asset will fall, relative to what is expected by the market and is reflected in the asset price. In this case, you could speculate by directly taking a short position in the asset. However, an alternative strategy could involve taking a short position on a derivative written on the asset. Taking a short position on a derivative could be more profitable, because futures and options can provide a form of leverage. The financial consequences are much larger, given the size of the initial investment.
- Finally, derivatives can be used to *exploit potential arbitrage opportunities*, which could arise, for instance, from small discrepancies

in the pricing of assets across different markets, or from inconsistency in the pricing of similar assets. Since arbitrage involves little or no risk, large amounts of resources have been devoted to developing software programmes that are able to execute automatic transactions whenever arbitrage opportunities are seen to arise.

Reading 1.4

Please now read Sections 1.6 to 1.9 of Hull (2018), pages 33–39. You should study carefully the examples in Tables 1.4 and 1.5, which illustrate the potential effect of leverage when using futures and options for speculative investment.

Hull (2018) Chapter 1, sections 1.6–1.9 in *Options, Futures, and Other Derivatives.*

Exercise 1.4

Please solve problem 1.9 on page 42 of the book by Hull.

Exercise 1.5

Please solve problem 1.16 on page 42 of the book by Hull.

Exercise 1.6

Please answer question 1.21 on page 42 of the book by Hull.

1.6 A ‘Health Warning’

While derivatives are very flexible instruments, if they are misused they can lead to potentially catastrophic losses. There has been a number of high-profile instances of reputable financial institutions being forced to fold because of the enormous losses suffered from speculation in the derivatives market. It is therefore essential that proper measures are put in place to monitor and control the risks associated with trading in derivatives. These measures include:

- setting proper risk limits
- monitoring traders
- separating trading execution from book keeping and accounting
- being aware of the usefulness but also of the limitations of quantitative models, and
- taking liquidity risk seriously.

These measures will not completely eliminate all risks, but can go a long way towards reducing the likelihood of negative outcomes from the trading in derivatives.

Reading 1.5

Please now read Section 1.10 and the Summary on pages 39–41 and Chapter 36, pages 828–39 of Hull (2018). Chapter 36 is towards the end of the book. You may not be able to understand yet all the references in the chapter, but it is very useful that you are aware of the potential issues in the use of derivatives, and of some of the main measures that should be taken in order to avoid the most serious adverse consequences; and you should therefore read this chapter now.

Hull (2018) Chapter 1, section 1.10, & Summary & Chapter 36 'Derivatives mishaps and what we can learn from them' in *Options, Futures*

1.7 Application: Data tables

In this section you will examine one of the tools used extensively in financial modelling, data tables, and you will create a data table to show the profit patterns from purchasing a call option on a stock. First you will examine a function that can be used to make your Excel workbooks more informative.

1.7.1 FORMULATEXT and Getformula

In this module you will develop and use relatively complex formulas and spreadsheets. To help understand the spreadsheets it would be useful to see the formulas contained in particular cells in a workbook. (Excel has the facility to Show Formulas in cells *instead* of the results, available on the Formulas tab, but it would be more useful to see the results *and* the formulas at the same time.)

This can be achieved using the Excel function FORMULATEXT. This function has the syntax =FORMULATEXT(reference), where 'reference' is the cell whose formula you would like to display. The function displays the formula that is contained in the reference cell, while leaving the results visible in the reference cell.

FORMULATEXT is a built-in Excel function, and it is available in new workbooks and any workbooks you create. For this reason it is probably most convenient for you to use FORMULATEXT in your own workbooks. However, FORMULATEXT was introduced in Excel 2013. If you are using an earlier version of Excel you can create a user-defined formula to do the same thing, and the instructions to do this are available in Benninga (2014), as follows.

Study Note 1.1

In the book by Benninga (2014) and the associated Excel workbooks you will see a formula called 'Getformula'. Getformula is a User-defined formula, and has the same effect as FORMULATEXT.

Advice on Getformula and the instructions to put Getformula into an Excel workbook are provided in Chapter 0 'Before all else' in Benninga (2014), Sections 0.2 to 0.6, pages 2–6. The code on page 4 is available to copy from the Excel file named 'FM4, Chapter 00, Before you start.xlsm', on the tab 'Getformula into VBA'.

To use the Visual Basic Editor you will need to put the Developer tab on your ribbon. If this is not present already, go to File>Options>Customize Ribbon and make sure the Developer tab is selected.

If you follow this approach to define the function, Getformula will now be available in the saved workbook, but it will not be available in new workbooks.

The workbook 'FM4, Chapter 00, Before you start.xlsm' already has Getformula defined, so you could use this file as the basis for your own workbooks, if you are unable to follow these instructions successfully.

As noted above, if you are using Excel 2013 or later it is probably more convenient to use the FORMULATEXT function in your own workbooks, and be aware that Getformula in Benninga (2014) and associated Excel workbooks performs the same function.

1.7.2 Data tables

Data tables are a powerful tool for analysing financial instruments, including derivatives, and have wide application in financial modelling. They are useful for conducting sensitivity analysis, analysing how the results vary as one or other inputs change. To demonstrate how to use a data table we will recreate the profit pattern for the call option shown in Figure 1.5 in Hull (2018). The profit pattern shows how the option profit changes for a range of prices of the underlying stock.

Intuitively, a data table works like this. For this example we need to create a formula for the profit from purchasing a call option on a stock. As you have seen in this unit, the profit will depend on:

- the premium paid to purchase the call option
- the strike price (also known as the exercise price)
- the price of the underlying stock at maturity.

You have seen that if the stock price at maturity is less than the strike price, the call option will not be exercised, the payoff will be zero, and the overall loss will be the call premium already paid. If the stock price is more than the exercise price, the option will be exercised. The payoff will be the positive difference between the stock price and the exercise price, and the overall profit will be reduced by the call premium already paid.

In Unit 2 you will examine in more detail the profits for various types of options, but based on the above reasoning we can say that the profit from a long call option is

$$\max\left(S_T - K, 0\right) - c \tag{1.3}$$

where:

S_T is the stock price at maturity, time T

K is the strike price

c is the premium paid for the call option.

If

$$S_T < K$$

the option would not be exercised,

$$\max\left(S_T - K, 0\right) = 0$$

and the overall loss equals −c.

If

$$S_T > K$$

the option will be exercised,

$$\max\left(S_T - K, 0\right) = S_T - K$$

and the overall profit is

$$\left(S_T - K\right) - c$$

The data table has two inputs. Firstly, the data table needs to reference the formula for the profit from the call option. Secondly, we need to specify a range of values for the stock price at maturity. When we have created the data table it will show the option profit for each of the values of the stock price we have specified.

We have described the intuition behind a one-dimensional data table – only one input is varying in the table, the stock price. We could extend the data table with additional formulas, to assess the impact of varying the stock price on, for example, the profit from writing a call option, purchasing a put option, and writing a put option. But the input being varied is always the stock price. Two-dimensional data tables allow you to assess the impact on one formula of varying two inputs. For example, we could assess the impact on the call option profit of various stock prices *and* strike prices.

Reading 1.6

Please now read from Chapter 31 'Data tables' in Benninga (2014), Sections 31.1–31.5 and Section 31.8, pages 823–26 and page 835. These sections explain how to set up a data table, using a relatively simple net present value example.

Benninga (2014) Chapter 31 'Data tables' in *Financial Modeling*.

As noted in Benninga, data tables can take up a lot of processing power. Excel recalculates formulas in a workbook automatically when there is a change in one of the cells on which the formulas are dependent. The Excel default recalculation setting is 'Automatic Except for Data Tables'. For the data tables to operate you will need to change the calculation option to 'Automatic' (Formulas>Calculation Options) or use the 'Calculate Now' button (also on the Formulas tab).

Review Question 1.1

The data table described in this last reading is available in the workbook 'FM4, Chapter 31, data tables.xlsm', in the tab 'Pages 823–26'.

Examine what happens to the cash flows, and the data table for NPV and IRR, if you change the cash flow in year 1 from 234 to 300. Hint: If the projected cash flows in years 1 to 7 change, but NPV and IRR in the data table do not change, use the 'Calculate Now' button on the 'Formulas' tab.

Let us now return to our example and create a chart for the call option profit pattern shown in Figure 1.5 in your Hull (2018). The strike price is $22.50, the call option premium is $1, and the illustrative range of stock prices at maturity is $15–$30. The formula for the call option profit requires a particular stock price at maturity – let us choose the current stock price, $20. The example in Figure 1.5 is for 2,000 options. To demonstrate the principles of creating a data table we will consider the profit from purchasing one call option.

The profit pattern has been created in the file 'M482 U1 Application Profit pattern long call option.xlsm'. Within this file:

- the terminal stock price is in cell B3
- the exercise price is in cell B5
- the call premium is in cell B7
- the call option profit formula, given by equation (1.3), is in cell B9.

The formula in B9 is shown using Getformula in cell C9, and FORMULATEXT in C10.

The call option profit using the values we have suggested is minus $1. The stock price of $20 is less than the strike price of $22.50, the call option is not exercised, but the call premium has been paid.

Now let us create the data table.

In the first row of the data table, the entry for the long call profit references the cell containing the formula for the call option profit, so the formula is '=B9'. (Note that if we wanted to consider 2,000 call options we could change this formula to '=2000*B9'.)

We place the range of terminal stock prices starting in the next row down, starting at $15 and going to $30, and we have chosen a step of $0.50.

To create the data table select the column of terminal stock prices *and* the formula referencing the call option profit formula, so cells C14:D45. Then go to Data>What-if Analysis>Data Table…

The range of terminal stock prices is arranged in a column. Leave the Row input cell blank. The column input cell is the terminal stock price in cell B3. (To conduct the What-if Analysis the data table will substitute each of the stock prices in the range for the contents of cell B3.)

If your Excel Calculation Option is set as 'Automatic Except for Data Tables', the call option profit for the range of terminal stock prices in the data table will be −1. Why is that? If this happens, refresh the data table using the

Calculate button. You should now have a range of call option profits starting at −1 for terminal stock prices $15 to $22.50, then −$0.50 rising to $6.50.

Now you can create the chart of the call option profit profile. The input range is the column of terminal stock prices and long call profit (*not* including the cells C14 and D14), and the chart type is 'Scatter with Straight Lines'.

Exercise 1.7

For the call option and the stock in Exercise 1.4 (question 1.9 on page 42 of Hull) create a data table and chart showing how the profits change for various values of the terminal stock price.

1.8 Conclusion

This unit has introduced the most common types of financial derivatives: forward contracts, futures and options. You have studied their different characteristics and have started to learn how you can use them to modify the payoff of your investment profile. You have also constructed simple profit patterns using data tables. In the next units of this module you will build on the basic concepts and techniques presented in this unit and you will learn how you can construct very complex risk profiles by making use of derivatives instruments.

1.9 Solutions to Exercises

Here are the answers to the exercises you've been assigned for this unit, but please don't check the answers until you've worked them out for yourself.

Exercise 1.1

a) At the end of the contract the investor sells 100,000 British pounds for US dollars at an exchange rate of 1.5000 US dollars per pound, whereas the spot exchange rate on maturity is 1.4900. The investor therefore gains 100,000 × (1.5000 − 1.4900) = $1,000.

b) At the end of the contract the investor sells 100,000 British pounds for US dollars at an exchange rate of 1.5000 US dollars per pound, when the spot exchange rate on maturity is 1.5200. The investor therefore loses 100,000 × (1.5200 − 1.5000) = $2,000.

Exercise 1.2

a) The trader commits herself to selling 50,000 pounds of cotton at the end of the contract at a price of 50 cents per pound. If the spot price of cotton at maturity is only 48.20 cents per pound, she will gain 50,000 × (0.5000 − 0.4820) = $900.

b) The trader must sell 50,000 pounds of cotton at the end of the contract at a price of 50 cents per pound, when the spot price of cotton at maturity is 51.30 cents per pound. She will therefore lose 50,000 × (0.5130 − 0.5000) = $650.

Exercise 1.3

You have sold a put option, and therefore you have a short position on a put contract. You have received the option premium, and have committed yourself to buying 100 shares at a price of $40, if the party who has purchased the put option decides to exercise her right to sell the shares at the exercise price. At present, it would not be profitable to exercise the option because the market price of each share is $41.

The investor who has purchased the put option will only find it profitable to exercise it if the stock price falls below $40. For instance, if the stock price falls to $30, the holder of the put option could purchase the shares at $30 and sell them to you at $40. She will have made a profit of 100 × (40 − 30) = $1,000, minus the price of purchasing the option contract. You will have to buy the shares at $40, although their market price is only $30. You will therefore have made a loss of $1,000, minus the premium that you have received from the option holder.

Exercise 1.4

One strategy that you can implement is to purchase 200 shares (5,800/29); the alternative strategy would be to purchase 2,000 call options (5,800/2.9). The table below illustrates the payoffs associated with each one of the two strategies when the stock price increases to $40 and when it declines to $25. When the stock price increases to $40, your profit from the first strategy will be 200 × ($40 − $29) = $2,200, and [2,000 × ($40 − $30)] − $5,800 = $14,200 from the second strategy, if you exercise your call option and purchase stocks at the strike price of $30. By contrast, when the stock price falls to $25 your losses will be 200 × ($29 − $25) = $800 from the first strategy, but will amount to your whole investment of $5,800 because you will have to let your options expire unexercised.

Table 1.1

Strategy	Stock price	
	$25	$40
Buy 200 shares	($800)	$2,200
Buy 2,000 call options	($5,800)	$14,200

You can verify that the second strategy yields larger gains when the stock price increases, but also larger losses when the stock price falls.

Exercise 1.5

The put option will be exercised against the trader if the stock price is less than $30 in December. If we consider the price that the trader has received for issuing the option, she will make a profit provided the stock price in December is greater than $26 (ignoring the time value of money).

Exercise 1.6

With futures and options, the gain to one party must always be equal to the losses suffered by the other party. Hence, if we add up the payoffs to both parties, they must always add up to zero. Graphically, the payoff from a short position is the mirror image of the payoff from a long position (that is, it is symmetrical about the horizontal axis).

Exercise 1.7

The initial share price is \$29. A call option with strike price \$30 costs \$2.90. With \$5,800 available to invest, it is possible to purchase 2,000 call options or 200 shares.

The formula for the profit from the call option is given by equation (1.3)

$$\max\left(S_T - K, 0\right) - c$$

and the formula for the profit from investing in shares is

$$S_T - S_0$$

where:

S_0 is the initial stock price.

The data table and chart showing the profit from two strategies, investing in the stock or the call option, for a range of stock prices at maturity of \$25 to \$40, is included in the file 'M482 Exercise 1.7 Profit pattern stock or long call.xlsm'. The chart is also shown in Figure 1.1.

Figure 1.1 Profits from purchasing shares or call options