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A SPECIFIC RULE IN INDIA FOR COMMON DIFFERENCE AS FOUND IN THE GOMMAṬASĀRA OF NEMICANDRA (c. 981)

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1. Introduction

Ācārya Nemicandra "Siddhānta Cakravartī" was a Digambara Jaina monk. He authored a number of works which became authoritative reference books for the Digambara Jaina tradition. The world famous colossal image of Bāhubalī was erected by his disciple Cāmuṇḍarāya, who has been a celebrated commander-in-chief and wise minister of the *Gaṇga* dynasty during the period from 953 to 985, at Śravaṇabeḷagoḷa in India. The first consecration ceremony of the statue was held on 13th March of 981. Nemicandra is said to have been in attendance there.[1]

The Jaina canons mainly deal with two systems. One is the system of *karma* where *karma* is the matter, exceptionally subtle, which actually does flow into the *jīva* (soul) and the other is the system of cosmology. Two treatises of Nemicandra's authorship are the *Gommaṭasāra* ('an essence ‹extracted from the previous sources on the *karma* system and composed› for *Gommaṭa* ‹i. e. Cāmuṇḍarāya›[2]') and the *Trilokasāra* ('essence of the three regions of the universe'). Both of them are post-canonical texts and written in Prakrit. The *Gommaṭasāra* deals with the *karma* system while the *Trilokasāra* deals with the Jaina system of cosmology and cosmography. The *Gommaṭasāra* has two sections: the *Jīvakāṇḍa* ('section regarding soul') and the *Karmakāṇḍa* ('section regarding *karma*'). The *Trilokasāra* is in only one section.

A lot of mathematical rules have been embedded by Nemicandra into these two treatises to apply them to solve the problems related with the respective systems. One of them is a specific rule offered by him in the *Gommaṭasāra* (*Karmakāṇḍa*) to find the common difference of an arithmetic progression. This rule is, as far as the present author knows, not found in any other treatise authored by either Nemicandra's predecessor or his successor. It remained unnoticed by historians of mathematics and will be brought into light and discussed

[1] Jadhav 2006: 75-81.

[2] Jadhav 1999: 19-24.

in this article for the first time. We shall also offer a rationale for it in the section four of this paper. Before we take up it, we need to introduce the terms related with arithmetic progression [A. P.] and offer a brief survey of the history of its development in ancient and medieval Indian mathematics prior to him.

If S be the sum of an A. P. of n terms, then

$$S = T_1 + T_2 + T_3 + ... + T_n \quad [1]$$

where

$$T_n = a + (n-1)d \quad [2]$$

where a and d are its first term and common difference respectively. It means $T_1 = a$.

We can write [1] as follows

$$S = a + (a+d) + (a+2d) + ... + [a+(n-2)d] + [a+(n-1)d] \quad [3]$$

or $$S = [a + a + a + ... n \text{ terms}] + [d + 2d + 3d + ... \ + (n-1)d]$$

or $$S = A + D \quad [4]$$

where

$$A = na \quad [5]$$

is called the sum of the first terms of the A. P. and

$$D = d + 2d + 3d + ... + (n-1)d \quad [6]$$

is called the sum of the common differences of the A. P.

Writing the A. P. [3] in reverse order, we have

$$S = [a + (n-1)d] + [a + (n-2)d] + ... + (a+2d) + (a+d) + a. \quad [7]$$

Adding [3] and [7], we get

$$S = n\left[a + \frac{(n-1)}{2}d\right]. \quad [8]$$

Indian interest in A. P. started quite early in the Vedic age. Quite a few instances are found in the *Taittirīya Saṃhitā,* the *Vājasaneyī Saṃhitā*, and other texts.[3] According to A. N. Singh, Indians must have obtained the formula for finding S at a very early date, but when exactly cannot be said for certain. It is, however, certain that in the period extending from 500 BCE to 400 BCE it was known, for in the *Bṛhaddevatā*, which is a summary of the deities and myths found in the *Ṛgveda* and is attributed to Śaunaka, we find $S = 500499$ for the A. P.: 2, 3, 4, …, 1000.[4] The rule for finding S in terms of a, d, n is given in the *Bakṣālī*

[3] Singh 1936: 607. Also see Datta &Singh 1993: 103f.

[4] Singh 1936: 608. Also see Datta &Singh 1993: 104. See also Macdonell 1904: Part I, *śloka* 3.130, 35 and Part II, 117.

(usually spelt *Bakhshali*) *Manuscript* (c. 400 or 7th century).[5] The rules on A. P. referred to by some prominent authors of early medieval India can be observed from Table I.

Table I

S. No.	The rule referred to			
	by the author	in the treatise	for	in terms of
1.	Āryabhaṭa I (born 476)	*Āryabhaṭīya*[6]	S	a, d, n
			S	a, T_n, n where T_n is the nth term or the last term (*antyadhana*) of an A. P.
		Āryabhaṭīya[7]	n	a, d, S
2.	Yativṛṣabha (some period between 176 and 609)	*Tiloyapaṇṇatti*[8]	S	a, d, n
		Tiloyapaṇṇatti[9]	d	a, n, S
		Tiloyapaṇṇatti[10]	n	a, d, S
		Tiloyapaṇṇatti[11]	S	a, d, n, i where i is an optional number (*iṭṭha*, Skt. *iṣṭa*).
		Tiloyapaṇṇatti[12]	S	a, d, n
3.	Brahmagupta (c. 628)	*Brāhma-sphuṭa-siddhānta*[13]	T_n	a, d, n
			M	a, T_n
			S	n, M

[5] Bag 1979: 13, 181; Hayashi 1985: 249.

[6] ĀB v. 19, p. 105.

[7] ĀB v. 20, p. 108.

[8] TP v. 2.76, p. 163; v. 2.81, p. 165.

[9] TP v. 2.84, p. 166.

[10] TP v. 2.85, p. 167; v. 2.86, p. 168.

[11] TP v. 2.64, p. 158; v. 2.70, p. 161; Jadhav 2005: 53-57.

[12] TP v. 2.65, p. 159; Jadhav & Shivakumar 2005: 47-50.

[13] BSS v. 17, p. 789.

				where M ($= (T_n + a) \div 2$) is its middle term (*madhyadhana*).
		Brāhma-sphuṭa-siddhānta[14]	n	a, d, S
4.	Śrīdhara (c. 799)	*Pāṭīgaṇita*[15]	S	a, d, n
			S	a, T_n, n
		Pāṭīgaṇita[16]	a	d, n, S
			d	a, n, S
		Pāṭīgaṇita[17]	n	a, d, S
		Triśatikā[18]	T_n	a, d, n
			M	a, T_n
			S	n, M
		Triśatikā[19]	a	d, n, S
			d	a, n, S
		Triśatikā[20]	n	a, d, S
5.	Mahāvīra (c. 850)	*Gaṇita-sāra-saṅgraha*[21]	S	a, d, n
		Gaṇita-sāra-saṅgraha[22]	A	a, n
			D	d, n
			S	A, D
		Gaṇita-sāra-saṅgraha[23]	T_n	a, d, n
			M	a, T_n
			S	n, M
		Gaṇita-sāra-saṅgraha[24]	n	a, d, S

[14] BSS v. 18, p. 797.

[15] PG v. 85, p. 110.

[16] PG v. 86, pp. 118-119.

[17] PG v. 87, p. 120.

[18] TŚ v. 39, p. 28.

[19] TŚ v. 40, p. 29.

[20] TŚ v. 41, p. 29.

[21] GSS v. 2.61, p. 45; v. 2.62, p. 46.

[22] GSS v. 2.63, p. 47.

[23] GSS v. 2.64, p. 48.

[24] GSS v. 2.69, p. 50; v. 2.70, p. 51.

		Gaṇita-sāra-saṅgraha[25]	d	S , A , n
			a	S , D , n
		Gaṇita-sāra-saṅgraha[26]	a	d , n , S
			d	a , n , S
		Gaṇita-sāra-saṅgraha[27]	d	a , n , S
		Gaṇita-sāra-saṅgraha[28]	a	d , n , S
6.	Āryabhaṭa II (c. 950 or sixteenth century)[29]	*Mahāsiddhānta*[30]	S	a , d , n
		Mahāsiddhānta[31]	a	d , n , S
		Mahāsiddhānta[32]	d	a , n , S
		Mahāsiddhānta[33]	n	a , d , S
7.	Nemicandra (c. 981)	*Trilokasāra*[34]	n	a , d , T_n
		Trilokasāra[35]	a	d , n , T_n
			T_n	a , d , n
			S	a , T_n , n
		Trilokasāra[36]	S	a , d , n

What is served in Table I from the *Trilokasāra* shows that Nemicandra (c. 981) was familiar with the rules on A. P.

[2] can be inverted as follows:

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[25] GSS v. 2.73, p. 52.

[26] GSS v. 2.74, p. 53.

[27] GSS v. 2.75, p. 53.

[28] GSS v. 2.76, p. 54.

[29] For his date see: Sewell 1924: preface, ix; Mercier 1993: 1-13; Pingree 1992: 56.

[30] MS v. 15.47, p. 158.

[31] MS v. 15.48, p. 158.

[32] MS v. 15.49, p. 158.

[33] MS v. 15.50, p. 159.

[34] TS v. 57 second hemistich, p. 51. Also see Datta 1935: 33.

[35] TS v. 163, p. 164. Also see Datta 1935: 32.

[36] TS v. 164, p. 165. Also see Datta 1935: 32.

$$a = T_n - (n-1)d \qquad [9]$$

Let us see how he sets forth both [2] and [9] in the *Trilokasāra*. They are stated as follows:

vegapadaṃ cayaguṇidaṃ bhūmimhi muhammi riṇadhanaṃ ca kae ।[37]

"Multiply the number of terms (*pada*, n) ‹of an A. P.› as subtracted by one by the common difference (*caya*, d). ‹The product when› subtracted from the last term (*bhūmi*, a) yields the first term (*muha*, Skt. *mukha*, a) and when added with the first term gives the last term ‹of the A. P.›."

Each of [2] and [9] can be inverted as follows:

$$d = \frac{T_n - a}{n-1}. \qquad [10]$$

Let us see how he sets forth [8] in the *Trilokasāra*. It is stated as follows:

padamegeṇavihīṇaṃ dubhājidaṃ uttareṇa saṃguṇidaṃ ।
pabhavajudaṃ padaguṇidaṃ padagaṇidaṃ taṃ vijāṇāhi ॥[38]

"The number of terms (*pada*, n) ‹of an A. P.› is subtracted by one and then divided by two and multiplied by ‹its› common difference (*uttara*, d). Add ‹the result to› the first term (*pabhava*, Skt. *prabhava*, a) and ‹then› multiply ‹the sum› by the number of terms (*pada*, n); know ‹the product› to be 'the sum of the ‹n› terms' (*padagaṇida*, Skt. *padagaṇita*, S) ‹of the A. P.›."

The context in which the above two verses were furnished by him in the first chapter, *lokasāmānyādhikāra* ('general chapter on the universe'), of the *Trilokasāra* was to discuss *indraka* (central hole) and *śreṇibaddhabilas* (holes arranged in ‹arithmetic› progression).[39] This cosmographic context has nothing do with the subject at hand and its background described in the *Gommaṭasāra* (*Karmakāṇḍa*). [8] can be inverted as follows:

$$d = \left(S \div (n-1)\frac{n}{2} \right) - \left(a \div \frac{(n-1)}{2} \right) \qquad [11]$$

or

$$d = \left(\frac{S}{n} - a \right) \div \frac{(n-1)}{2} \qquad [12]$$

[37] TS v. 163 first hemistich, p. 164.

[38] TS v. 164, p. 165.

[39] TS vv. 150-177, pp. 157-82.

or $$d = (S - A) \div \frac{(n^2 - n)}{2}$$ [13]

or $$d = \left(\frac{2S}{n} - 2a\right) \div (n-1)$$ [14]

The rule for d in the shape of [11] was known to Yativṛṣabha.[40] It was known in the shape of [12] to Śrīdhara[41], Mahāvīra[42] and Āryabhaṭa II[43]. Mahāvīra knew it in the shape of [13] and [14] as well.[44]

This brief survey on A. P. enables us to assert that the rule, each of [10] and [11]/[12]/[13]/[14], for d must have been known to Nemicandra in one or the other shape. This was the very purpose of the survey.

2. The installation of the specific rule

In the *Gommaṭasāra* (*Karmakāṇḍa*) he sets forth a specific rule to find d as stated below:

> *ubhayadhaṇe saṃmilide padakadiguṇasaṃkharūvahadapacayaṃ* ।
> *savvadhaṇaṃ taṃ tamhā padakadisaṃkheṇa bhājide pacayaṃ* ।।[45]

"Both (*ubhaya*) ‹'the sum of the first terms', A, and 'the sum of the common differences', D,› when added happen to be ‹equal to› the square of 'the number of terms' (*pada*, n) as multiplied by an ‹arbitrary› number (*saṃkha*, Skt. *saṅkhya*, k (say)) and by the common difference (*pacaya*, Skt. *pracaya*, d). ‹Therefore,› the sum (*savvadhaṇa*, Skt. *sarvadhana*, S) being divided by the square of 'the number of terms' (*pada*, n) and by an ‹arbitrary› number (*saṃkha*, Skt. *saṅkhya*, k) gives rise to the common difference (*pacaya*, Skt. *pracaya*, d)."

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40 TP v. 2.84, p. 166.

41 PG v. 86 second hemistich, p. 119; TŚ v. 40 second hemistich, p. 29.

42 GSS v. 2.74 second hemistich, p. 53.

43 MS v. 15.49, p. 158.

44 GSS v. 2.73 first hemistich, p. 52 and v. 2.75, p. 53 respectively.

45 GSK$_1$ v. 902, p. 1252, cf. GSK$_2$ v. 902, p. 287.

That is to say:

$$A + D = n^2 kd \quad [15]$$

or $S = n^2 kd$

Therefore, $$d = \frac{S}{n^2 k}. \quad [16]$$

The first hemistich of the above verse contains [15] while [16] is embedded into its second hemistich. Since [16] contains k, it cannot be a general rule. It is required to be a specific rule. It enables us to find d when only S and n are known. Unlike [10] and [11]/[12]/[13]/[14] it is free from a. It is found nowhere in Table I. It has not been a part of the mainstream of the mathematics in India concerned with A. P.

3. The context of the rule

If each term of [1] is detached into an A. P. of m terms, then

$$T_n = t_{n.1} + t_{n.2} + t_{n.3} + ... + t_{n.m} \quad [17]$$

where

$$t_{n.m} = \alpha_n + (m-1)\delta \quad [18]$$

where α_n and δ are its first term and common difference respectively. It means $t_{n.1} = \alpha_n$.

We can write [17] as follows

$$T_n = \alpha_n + (\alpha_n + \delta) + (\alpha_n + 2\delta) + ... + (\alpha_n + (m-1)\delta).$$

Then

$$\Delta = \delta + 2\delta + 3\delta + ... + (m-1)\delta \quad [19]$$

is called the sum of the common differences of the A. P. [17].

The context in which Nemicandra set forth [16] runs into sixteen verses, namely from 897 to 912 including the one stated above in the section two, of the chapter eight of the *Gommaṭasāra* (*Karmakāṇḍa*) (GSK). It is as follows:[46]

Thought-activity (*karaṇa*), in which a soul's pure thoughts increase infinite fold at every instant (*samaya*), is a special process of thought-concentration.[47] It is the instrumental cause for destruction (*kṣapaṇa*) or suppression (*upaśamana*) of twenty-one sub-classes of conduct-deluding (*cāritra-moha*) *karma*.[48] Among those twenty-one sub-classes are four partial-vow-preventing passions (*apratyākhyānāvaranīya kaṣāya*), four total-vow-preventing

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[46] Common Sanskrit terms are used instead of the original Prakrit.

[47] GSJ, Sital Prasad's comments below verse 48, p. 38.

[48] GSK_2 v. 897 first hemistich, p. 285.

passions (*pratyākhyānāvaranīya kaṣāya*), four perfect-conduct-preventing passions (*saṃjvalana kaṣāya*) and nine quasi-passions (*nokaṣāya*, slight or minor passions). Each of the first three contains anger (*krodha*), pride (*māna*), deceit (*māyā*) and greed (*lobha*). And the nine quasi-passions are laughter (*hāsya*), indulgence (*rati*), ennui (*arati*), sorrow (*śoka*), fear (*bhaya*), disgust (*jugupsā*), feminine inclination (*strī-veda*), masculine inclination (*puṃ-veda*) and neither feminine nor masculine sexual inclination (*napuṃsaka-veda*).[49] Thought-activity is divided into three kinds. They are (1) the lower-thought-activity (*adhaḥ pravṛtta karaṇa*), (2) the new-thought-activity (*apūrva karaṇa*) and (3) the advanced-thought-activity (*anivṛtti karaṇa*).

(1) The lower-thought-activity is named so because using it the quality (*bhāva*) of a posterior soul may grow to be as pure as that of a prior soul. In other words, due to more extensive practice, a soul who has commenced purifying thoughts later may come up to the level of the soul who commenced the same earlier. In mathematical terms, on the path leading to purifying thoughts the rate of progress of a posterior soul may be higher than that of a prior soul. The lower-thought-activity is used by a soul in the perfect vow-stage (*apramatta virati*) which is the seventh of the fourteen *guṇasthāna*s (qualitative stages of spiritual development), in which the embodied soul has all vows and keeps them perfectly. The lower thought-activity is performed not longer than one *antara-muhūrta*, that is, 48 minutes minus one *samaya* where *samaya* (instant) is an indivisible part of time. The increase of pure thoughts is theoretically calculated in terms of a uniform progression (*sadṛśavṛddhi*) (i. e., in A. P.).[50]

(2) Having passed the *antara-muhūrta* stage in the lower-thought-activity the soul is engrossed in the new-thought-activity, associated with the eighth *guṇasthāna*, where thoughts that had not arisen before arise. The duration of the new-thought-activity also is one *antara-muhūrta*. In the stage of new-thought-activity, if the souls commence purifying thoughts at the same instant, their progress onwards may be equal or unequal; but none of them can ever be overtaken by any soul who commences afterwards.[51]

(3) In the advanced-thought-activity, associated with the ninth *guṇasthāna*, the souls that commenced purifying thoughts at the same instant shall continue to go forward

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[49] For twenty-one sub-classes of conduct-deluding *karma* see Jaini 1918: 132f.

[50] GSK_2 vv. 897-899 and Sital Prasad's comments, pp. 285-7. For verses 47, 48 and 49 of the GSJ being identical with verses 897, 898 and 899 of the GSK see GSJ vv. 47-99 and Jaini's comments, pp. 37-44. For the meaning of *bhāva* see Jaini 1918: 33. For a brief account of spiritual stages see GSJ vv. 1-69, pp. 1-51. For *antara-muhūrta* see Jaini 1918: 17. The expression 'pure thoughts' refers to 'the number of pure thoughts.' The explanation found in the commentary on the GSK for the latter is *viśuddhi-kaṣāya-pariṇāma*, 'the number of passions that are purified out or removed out' if translated literally.' See GSK_1 below v. 899, p. 1250.

[51] GSJ vv. 50-53, pp. 39f. and 44, and GSK_2, p. 286. Verse 50 of GSJ is identical with verse 908 of GSK.

uniformly without any difference in the degree of purity. Here all the twenty-one sub-classes of conduct-deluding *karma* are destroyed or suppressed by the soul.[52]

In order to demonstrate the lower-thought-activity Nemicandra assumed[53]

$$\left.\begin{array}{l} S = 3072 \text{ (total number of thoughts that are to attain purity)} \\ n = 16 \text{ (number of instants at each of which purity is to be attained)} \\ k = 3 \text{ (arbitrary number chosen)} \\ \text{and calculated that} \\ d = 4 \text{ (the rate of progress in purity)} \\ m = 4 \text{ (number of the sub-terms of each term)} \\ \delta = 1 \text{ (common difference among the sub-terms).} \end{array}\right\} \quad [20]$$

Here n is said to be the number of vertical terms (*ūrdhvādhvāna*), d the vertical common difference (*ūrdhvaviśeṣa*), m the number of horizontal terms (*tiryagadhvāna*), and δ the horizontal common difference (*tiryagviśeṣa*).[54] See, in view of these data, Table II.

After the above assumption and calculation he sets forth those rules that he used to calculate d, m, δ and some other intermediates including D and a.

Firstly, he states that, according to his predecessors, in the lower-thought-activity D is numerable part of A.[55] That is to say: if

$$D = A \div \lambda, \quad [21]$$

λ, in the light of the above data, comes to be $27/5$. This result cannot be arrived at without knowing the rules for finding D and a. He incorporates them into the latter verses.

Secondly, he composes the rules for [15] and [16] into the verse which we have already noticed above in the section two.

Thirdly, for finding a he enunciates the formula,

$$(S - D) \div n = a, \quad [22]$$

in the first hemistich of the following verse.

cayadhaṇahīṇaṃ davvaṃ padabhajide hodi ādiparimāṇaṃ ।
ādimmi caye uḍḍhe paḍisamayadhaṇaṃ tu bhāvāṇaṃ ॥[56]

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[52] GSJ vv. 54-57, pp. 40f. and 44, and GSK$_2$, p. 286. Verse 56 of GSJ is identical with verse 911 of GSK.

[53] GSK$_2$ v. 900 and its translation by Sital Prasad and his comments, pp. 287f.

[54] Nemicandra, *Gommaṭasāra* (*Karmakāṇḍa*) (Ed. Upadhye and Shastri), v. 900 and its commentary, pp. 1250f.

[55] GSK$_2$ v. 901, p. 287.

"The value (*parimāṇa*) of the first term (*ādi*, a) is arrived at when 'the sum of the common differences' (*cayadhaṇa*, Skt. *cayadhana*, D) is subtracted from the sum (*davva*, Skt. *dravya*, total number of thoughts that are to attain purity, S) and then divided by 'the number of terms' (*pada*, n). The ‹number of› thoughts (*bhāva*) ‹that attained purity› at each instant (*samaya*) is obtained by adding the common difference (*caya*, d) ‹in succession› to ‹this› first term (*ādi*, a)."

In the second hemistich of the above verse he instructs how to prepare the required A. P. That A. P. can be seen in the first column of Table II. To find a using [22] it is essential that D must be known at the outset.

Fourthly, in order to educate how to calculate D he, equating the corresponding terms given in the right hand side of [6] with the formula [8], sets forth a rule as follows:

pacayadhaṇassāṇayaṇe pacaya pabhavaṃ tu pacayameva have ।
rū^ūṇapadaṃ tu padaṃ savvatthavi hodi ṇiyameṇa॥[57]

"In order to calculate 'the sum of the common differences' (*pacayadhaṇa*, Skt. *pracayadhana*, D) ‹of an A. P.›, the common difference (*pacaya*, Skt. *pracaya*, d), the first term (*pabhava*, Skt. *prabhava*, a) ‹of the common differences›, which is the same as the common difference (*pacaya*, Skt. *pracaya*, d) is, and 'the number of terms less one' (*rū^ūṇapada*, Skt. *rūponapada*, $(n-1)$) ‹to be assigned› to 'the number of terms' (*pada*, n) are ‹always taken› according to the rule (*ṇiyama*, Skt. *niyama*)."

That is to say: in order to calculate D,

$d \to d$, $d \to a$, and $(n-1) \to n$.

Accordingly,

$$D = \frac{1}{2}n(n-1)d . \qquad [23]$$

It may easily be understood that [22] is arrived at when [23] is subtracted from [8] and then divided by n. Using [16] d, in the light of $S = 3072$, $n = 16$, $k = 3$, comes to be 4. Then, using [23], $D = 480$. Subsequently, using [22], $a = 162$. By adding 4 to 162 in

[56] GSK$_2$ v. 903, p. 287.

[57] GSK$_2$ v. 904, p. 289.

succession we get other terms of thoughts that attained purity. See the column one in Table II. In the later part of the context he talks about the sub-terms of thoughts.

Fifthly, he states that

$$m = n \div \mu \qquad [24]$$

where μ is an arbitrary number. Here the terms used by him for m and n are *aṇukaṭṭipada* (Skt. *anukṛṣṭipada*, 'number of the terms (i. e. thoughts) that are ploughed along' if literally translated) and *savvaddhāṇa* (Skt. *sarvādhvāna*) respectively.[58] Since it is noticeable from [20] that $m = 4$ and $n = 16$, it can be easily inferred that he assumed $\mu = 4$. This is why each term of the vertical A. P. is detached into four sub-terms. See Table II.

Sixthly, he states

$$\delta = d \div m \qquad [25]$$

in the first hemistich of the following verse.

> *aṇukaṭṭipadeṇa hade pacaye pacayo du hoi tericche* ।
> *pacayadhaṇūṇaṃ davvaṃ sagapadabhajidaṃ have ādī* ।।[59]

"The ‹vertical› common difference (*pacaya*, Skt. *pracaya*, d) when divided by the number of horizontal terms (*aṇukaṭṭipada*, Skt. *anukṛṣṭipada*, 'number of the terms (i. e. thoughts) that are ploughed along' if literally translated, m) gives rise to the horizontal (*tericche*, Skt. *tirśchi*) common difference (*pacaya*, Skt. *pracaya*, δ). 'The sum of the ‹horizontal› common differences' (*pacayadhaṇa*, Skt. *pracayadhana*, Δ) subtracted from the ‹horizontal› sum (*davva*, Skt. *dravya*, T_n) gives rise to the first ‹horizontal› term (*ādi*, α_n) when divided by 'the own ‹number of› terms' (*sagapada*, Skt. *svakapada*, m)."

The second hemistich of the above verse contains

$$(T_n - \Delta) \div m = \alpha_n . \qquad [26]$$

Δ is the prerequisite of [26] to find α_n. It can be found from [19] in the same manner in which D was found using [23]. So,

$$\Delta = \frac{1}{2} m(m-1)\delta . \qquad [27]$$

Let us see how to calculate α_1. Using [25] δ, in the light of $d = 4$, $m = 4$, comes to be 1. Then, using [27], $\Delta = 6$. Subsequently, using [26], $\alpha_1 = 39$ as $T_1 = 162$.

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[58] GSK$_2$ v. 905, p. 290.

[59] GSK$_2$ v. 906, p. 290.

Seventhly and finally, he states that δ is successively added to each term commencing from α_n. Like this, a vertical and horizontal assortment (*uḍḍhtiriyarayaṇā*, Skt. *ūrdhvatiryagracanā*) should be known in the lower-thought-activity.[60] See Table II.

Table II

Term		Sub-terms							
T_n (number of thoughts that attained purity in each instant)		$t_{n.1}$ (1st division)		$t_{n.2}$ (2nd division)		$t_{n.3}$ (3rd division)		$t_{n.4}$ (4th division)	
T_1	162	$t_{1.1}$	39	$t_{1.2}$	40	$t_{1.3}$	41	$t_{1.4}$	42
T_2	166	$t_{2.1}$	40	$t_{2.2}$	41	$t_{2.3}$	42	$t_{2.4}$	43
T_3	170	$t_{3.1}$	41	$t_{3.2}$	42	$t_{3.3}$	43	$t_{3.4}$	44
T_4	174	$t_{4.1}$	42	$t_{4.2}$	43	$t_{4.3}$	44	$t_{4.4}$	45
T_5	178	$t_{5.1}$	43	$t_{5.2}$	44	$t_{5.3}$	45	$t_{5.4}$	46
T_6	182	$t_{6.1}$	44	$t_{6.2}$	45	$t_{6.3}$	46	$t_{6.4}$	47
T_7	186	$t_{7.1}$	45	$t_{7.2}$	46	$t_{7.3}$	47	$t_{7.4}$	48
T_8	190	$t_{8.1}$	46	$t_{8.2}$	47	$t_{8.3}$	48	$t_{8.4}$	49
T_9	194	$t_{9.1}$	47	$t_{9.2}$	48	$t_{9.3}$	49	$t_{9.4}$	50
T_{10}	198	$t_{10.1}$	48	$t_{10.2}$	49	$t_{10.3}$	50	$t_{10.4}$	51
T_{11}	202	$t_{11.1}$	49	$t_{11.2}$	50	$t_{11.3}$	51	$t_{11.4}$	52
T_{12}	206	$t_{12.1}$	50	$t_{12.2}$	51	$t_{12.3}$	52	$t_{12.4}$	53
T_{13}	210	$t_{13.1}$	51	$t_{13.2}$	52	$t_{13.3}$	53	$t_{13.4}$	54
T_{14}	214	$t_{14.1}$	52	$t_{14.2}$	53	$t_{14.3}$	54	$t_{14.4}$	55
T_{15}	218	$t_{15.1}$	53	$t_{15.2}$	54	$t_{15.3}$	55	$t_{15.4}$	56
T_{16}	222	$t_{16.1}$	54	$t_{16.2}$	55	$t_{16.3}$	56	$t_{16.4}$	57

The following is an explanation offered by Sital Prasad regarding the horizontal terms shown in Table II:

> "Let us assume that 4 persons have entered upon a stage of lower thought-activity one having 39, the other 40, the 3rd 41 and the 4th 42 steps in thought purity. Each is advancing every moment by one step. In the next instant the four will respectively have progressed to 40, 41, 42 and 43 steps. Then suppose

[60] GSK$_2$ v. 907, p. 290.

that another set of four persons have entered upon such thought purity. They will have in the first instant, steps of 39, 40, 41 and 42. Here the person who has 40 in the first instant will be equal to that person who has 40 in the 2nd instant, and one who has 41 in the first instant will be equal to the person who has 41 in the 2nd instant. The one in the first group who has 39 in the first instant will have 42 in the 4th instant while the 4th person of the 2nd group has 42 in the first instant. Thus a person entering upon thought-purity later may be equal to one who has commenced earlier. Where such progress of increase of purity is possible, it is called the lower thought-activity."[61]

Because of having common characteristics the thoughts of the last three divisions in the first instant respectively match those of the first three divisions in the second instant. For the same reason the thoughts of the last three divisions in the second instant respectively match those of the first three divisions in the third instant and the same follows in the consecutive instants. Only the thoughts of the first division in the first instant and those of the last division in the last or sixteenth instant remain matchless. It means that the thoughts that are to appear to attain purity in the posterior instants are partly covered in the prior instants. This seems to be the plausible interpretation of *aṇukaṭṭipada* (Skt. *anukṛṣṭipada*, 'number of the terms (i. e. thoughts) that are ploughed along' if literally translated) in the lower-thought-activity. On the other hand, in the new-thought-activity $m = 0$ for at each instant innumerable number of new thoughts attain purity.[62] In the advanced-thought-activity one thought per instant attains purity.[63]

As far as the lower-thought-activity is concerned, it was known prior to Nemicandra in Jaina philosophy but the way in which he demonstrated it using mathematics, especially the rule [16], is not found in any treatise anterior to the *Gommaṭasāra*.[64]

4. Rationale for the rule

We have noticed above that there are three formulae that contain arbitrary numbers. One is [16] that contains k. The others are [`21] and [24] that contain λ and μ respectively. Both

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[61] GSK_2, p. 292.

[62] GSK_1 v. 910, p. 1268.

[63] GSK_1 v. 912, p. 1272.

[64] Dhavalā, p. 181.

of them simply correspond to the ratio. k too may be regarded to be a ratio between S and n^2d. But it is not usually expected that when one wants a ratio, he will hit upon such a consequent that contains the product of two terms, namely n and d, and one of them is with its square, namely n^2, while its antecedent contains a single term, namely S. For this reason k seems to have been involved passing through some process on d, S and n. [25] cannot be compared with [16] for the former is a result of the ratio simply taken.

Now the question is how Nemicandra processed to hit on [16]. [15], especially its left hand side, does seem to be a clue in this matter. He serves it as a prior step to [16]. Using this clue we can suggest a rationale for [16]. Our rationale is as follows:

$$S = A + D$$

or $$S = na + \frac{1}{2}n(n-1)d$$

or $$S = \frac{1}{2}dn^2 + \left(a - \frac{d}{2}\right)n. \qquad [28]$$

d can never be zero but $(a-(d/2))$ may be zero as S is a quadratic expression in n. On this ground we are able to deduce that

$$S \propto dn^2$$

or $$S = kdn^2 \qquad [29]$$

where $k(\neq 0)$ is an arbitrary number, which, when inverted, gives [16].

Although it is true that Indians had a sound knowledge of quadratic equations and the methods of their solutions by his time[65], we do not have, in fact we could not find, any evidence, especially that $S \propto dn^2$ was known, to substantiate that the method employed in our rationale may have been literally used by Nemicandra. However, our rationale does suggest that [16] is a rule, certainly of specific nature, for finding d as it can be obtained by way of processing on d, S and n.

5. Relevance of the rule

To determine d one needs the value of k besides S and n, and when one does select the value of k then d is calculated using [16] and thereafter a is calculated using [23] for the intermediate purpose and [22] for the purpose. It means that k is not only directly related with the computation of d but also ultimately determines a. In this sense too [16] is specific.

For other values of k than 3 we shall have a variety of assortments. For the reason that the formulae [24] and [25] are not derived using any process, our mathematical interest

[65] Datta & Singh 1938: 59-75. See also PG v. 87, p. 120 and K. S. Shukla's translation and comment, pp. 70f.

does not lie in the formation of the horizontal terms (i. e. the sub-terms of thoughts). It does, because of [16], lie in the formation of the vertical terms. Table III contains various arithmetic progressions in accordance with various values of k while $S(=3072)$ and $n(=16)$ are fixed. In the context of the above sort, k would not take any negative number. On the other hand, we have assigned negative numbers to k in Table III so that we can view a larger use of [16]. For the same purpose [11]/[12]/[13]/[14], which needs a to determine d, may be used but in that case we shall have to choose the value for a. For the employment of [10] we shall have to choose not only a but also T_n. On the other hand, Nemicandra seems to have "fed two birds", d and a, "with one scone", k, that too in a methodical manner. Hence [16] can be applied to a system similar to the lower-thought-activity.

Table III

T_n	A. P. when $k =$							
	-4	-3	-2	-1	1	2	3	4
T_1	214.5	222	237	282	102	147	162	169.5
T_2	211.5	218	231	270	114	153	166	172.5
T_3	208.5	214	225	258	126	159	170	175.5
T_4	205.5	210	219	246	138	165	174	178.5
T_5	202.5	206	213	234	150	171	178	181.5
T_6	199.5	202	207	222	162	177	182	184.5
T_7	196.5	198	201	210	174	183	186	187.5
T_8	193.5	194	195	198	186	189	190	190.5
T_9	190.5	190	189	186	198	195	194	193.5
T_{10}	187.5	186	183	174	210	201	198	196.5
T_{11}	184.5	182	177	162	222	207	202	199.5
T_{12}	181.5	178	171	150	234	213	206	202.5
T_{13}	178.5	174	165	138	246	219	210	205.5
T_{14}	175.5	170	159	126	258	225	214	208.5
T_{15}	172.5	166	153	114	270	231	218	211.5
T_{16}	169.5	162	147	102	282	237	222	214.5
$S =$	3072	3072	3072	3072	3072	3072	3072	3072

[29] seems to have been first obtained as both the entire verse referred to by Nemicandra for [16] and our rationale suggest. We can use [29] to generate various arithmetic progressions by finding S in accordance with various values of k while n and d remain fixed. See Table IV.

Table IV

T_n	If $n(=16)$ and $d(=4)$ are fixed, A. P. when $k =$							
	-4	-3	-2	-1	1	2	3	4
T_1	-286	-222	-158	-94	34	98	162	226
T_2	-282	-218	-154	-90	38	102	166	230
T_3	-278	-214	-150	-86	42	106	170	234
T_4	-274	-210	-146	-82	46	110	174	238
T_5	-270	-206	-142	-78	50	114	178	242
T_6	-266	-202	-138	-74	54	118	182	246
T_7	-262	-198	-134	-70	58	122	186	250
T_8	-258	-194	-130	-66	62	126	190	254
T_9	-254	-190	-126	-62	66	130	194	258
T_{10}	-250	-186	-122	-58	70	134	198	262
T_{11}	-246	-182	-118	-54	74	138	202	266
T_{12}	-242	-178	-114	-50	78	142	206	270
T_{13}	-238	-174	-110	-46	82	146	210	274
T_{14}	-234	-170	-106	-42	86	150	214	278
T_{15}	-230	-166	-102	-38	90	154	218	282
T_{16}	-226	-162	-98	-34	94	158	222	286
$S =$	-4096	-3072	-2048	-1024	1024	2048	3072	4096

[29] can be inverted as follows:

$$n = \sqrt{\frac{S}{kd}} \qquad [30]$$

It can be used to generate various arithmetic progressions by finding n in accordance with various appropriate values of k while S and d remain fixed. See Table V.

6. Concluding remarks

There seems to have been two rules before Nemicandra for finding d. One rule [10] is in terms of a, n, T_n. It could not be utilized by him for in the lower-thought-activity neither a, the number of thoughts that are to attain purity in the first instant, nor T_n, that of thoughts that are to attain purity in the nth or last instant, is predetermined. The other rule was [11]/[12]/[13]/[14] or the like. [11]/[12]/[14] is in terms of a, n, S while [13] is in terms of A, n, S. These variants too could not be employed by him, for in the lower-thought-activity both n, the number of instants at each of which purity is to be attained by thoughts, and S,

Table V

T_n	If $S(=-3072)$ and $d(=4)$ are fixed, A. P. when			If $S(=3072)$ and $d(=4)$ are fixed, A. P. when		
	$k=-3$	$k=-12$	$k=-48$	$k=3$	$k=12$	$k=48$
T_1	-222	-398	-774	162	370	762
T_2	-218	-394	-770	166	374	766
T_3	-214	-390	-766	170	378	770
T_4	-210	-386	-762	174	382	774
T_5	-206	-382	S = -3072	178	386	S = 3072
T_6	-202	-378		182	390	
T_7	-198	-374		186	394	
T_8	-194	-370		190	398	
T_9	-190	S = -3072		194	S = 3072	
T_{10}	-186			198		
T_{11}	-182			202		
T_{12}	-178			206		
T_{13}	-174			210		
T_{14}	-170			214		
T_{15}	-166			218		
T_{16}	-162			222		
S =	-3072			S = 3072		

the total number of thoughts that are to attain purity, are fixed, but a is not predetermined. Since, in [16], d is inversely proportional to k, d will increase when k decrease. We are able to observe this if we go by Table III. It also shows that the values of a decrease when those of k decrease. It is k which maintains d and a in the above manner. A rule having this sort of feature was required for the demonstration of the lower-thought-activity, especially its important facet that the rate of progress of a posterior soul may be higher than that of a prior soul on the path leading to purifying thoughts. And for that particular purpose [16] was created in mathematics for the service of Jaina philosophy.

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