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JAINA VERSUS BRAHMANICAL MATHEMATICIANS*

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Probably the earliest text that provides us first-hand information about classical Indian geometry in practice is Bhāskara's commentary on Āryabhaṭa's *Āryabhaṭīya*, the *Āryabhaṭīya Bhāṣya*. The *Āryabhaṭīya* dates from 499 or 510 CE, depending on whether one follows Pingree's or Billard's dating of this text.[1] Bhāskara's *Āryabhaṭīya Bhāṣya* on it dates from 629 CE. Both Āryabhaṭa's text and Bhāskara's commentary on it deal in part with geometry, and present a number of geometrical theorems. Bhāskara's *Bhāṣya* also provides diagrams and exercises. However, neither of these two texts provides proofs for their theorems. Modern scholars have sometimes tried to explain this by assuming that proofs were handed down orally, and were not written down.[2] This assumption is demonstrably incorrect, at least for this case. Both these texts contain some theorems that are false. They were false in the form they were given by Āryabhaṭa around the year 500 CE, and they were still false in Bhāskara's text in 629 CE. They had apparently been handed down in this erroneous form for some 130 years, and no one during these 130 years had ever noticed that they were mistaken. This would have been impossible if generations of students had bent over the proofs of these theorems. Discovering the mistakes would have been a matter of routine, and Bhāskara's *Bhāṣya* would not have repeated incorrect theorems.

[Āryabhaṭa is wrong where he gives the volume of a pyramid as:[3] "Half the product of the height and the [surface of the triangular base] is the volume called 'pyramid'." The correct volume of a pyramid is a third, not half, of the product here specified. In spite of this,

* This paper draws heavily on Bronkhorst 2001.

[1] "Tradition places Āryabhaṭa (born A.D. 476) at the head of the Indian mathematicians and indeed he was the first to write formally on the subject" (Kaye 1915: 11).

[2] For the question of mathematical proofs in ancient traditions, see Chemla 2012.

[3] *Āryabhaṭīya Gaṇitapāda* 6cd: *ūrdhvabhujātatsaṃvargārdhaṃ sa ghanaḥ ṣaḍaśrir iti.* Cf. Keller 2006 I: 30; II: 27; Bag 1979: 159.

Bhāskara accepts Āryabhaṭa's rule and carries out some (incorrect) calculations with its help. The same is true of Āryabhaṭa's incorrect rule for the volume of a sphere.[4]]

It is perhaps not surprising that Bhāskara was not critical with regard to the text he had received from Āryabhaṭa. In his comments on the first chapter he states, for example, that all knowledge derives from Brahmā; Āryabhaṭa pleased Brahmā on account of his great ascetic practices and could then compose, for the well-being of the world, the ten *Gītikāsūtra*s on planetary movement (chapter 1), as well as the one hundred and eight *āryā* verses on arithmetic (ch. 2), time (ch. 3) and the celestial globe (ch. 4).[5] Elsewhere (p. 189 l. 14-15) he attributes superhuman qualities to Āryabhaṭa, calling him *atīndriyārthadarśin* "seeing things that are beyond the reach of the senses". In the third chapter, the *Kālakriyāpāda*, moreover, Bhāskara cites, under verse 5, a verse from an earlier (non-identified) astronomical text — about the months in which the sun passes the summer and winter solstices — and calls it Smṛti, more precisely: *our* Smṛti (p. 182 l. 9: *asmākaṃ smṛtiḥ*). All this confirms that Bhāskara handed down a tradition that he looked upon as authoritative and more or less sacred, and not a body of doctrines that could be improved upon by critical reflection. Clearly, independent critical reasoning, with the possible result that a respected authority is shown to be wrong, could have no place in his work.

We can contrast this with what we know about the history of Indian philosophy. Indian philosophers were not indifferent about proofs. They used proofs in their treatises, and theorized about it. Clear ideas about what constitutes valid proof were well in place at the time when Bhāskara wrote his *Āryabhaṭīya Bhāṣya*. The question is therefore, why was Bhāskara not interested?

To find an answer, we must stay a little longer with the philosophers. Their interest in proofs and in logic in general has to be understood in the specific circumstances in which Indian philosophy developed. The history of Indian philosophy can to at least some extent be described as the history of different schools of thought that confront each other. The opposition between Buddhists and Brahmins is particularly important here, but it is not the

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4 Cp. *Āryabhaṭīya Gaṇitapāda* 7: *samapariṇāhasyārdhaṃ viṣkambhārdhahatam eva vṛttaphalam / tannijamūlena hataṃ ghanagolaphalaṃ niravaśeṣam //* "Half the even circumference multiplied by half the diameter is precisely the fruit (i.e., the area) of a circle. That (the area) multiplied by its own square root is the exact volume (lit. the without-a-remainder solid fruit) of a sphere" (tr. Hayashi 1997: 198; similarly Clark 1930: 27; Keller 2006 I: 33-35; II: 32-33). Bhāskara also provides an approximate, "practical" (*vyāvahārika*), rule for calculating the volume of a sphere (p. 61 l. 27): *vyāsārdhaghanaṃ bhittvā navaguṇitam ayoguḍasya ghanagaṇitam*, "Having divided [into two] the cube of half the diameter multiplied by nine, the calculation of the volume of an iron ball [has been carried out]."

5 *Āryabhaṭīya Bhāṣya* p. 11 l. 23 - p. 12 l. 1: *anenācāryeṇa mahadbhis tapobhir brahmārādhitaḥ / ... / ato 'nena lokānugrahāya sphuṭagrahagatyarthavācakāni daśa gītikāsūtrāṇi gaṇitakālakriyāgolārthavācakam āryāṣṭaśatañ ca vinibaddham /.*

only one. Philosophers tried to defend their own positions and show that the positions of their opponents were incoherent or worse. In these confrontations — which may sometimes have taken the shape of debates, sometimes public debates — logic and proofs were vital.

Indian mathematicians at the time of Bhāskara did not normally find themselves in a situation of confrontation. They did not normally have to prove that others were wrong and only they themselves right. Debates about mathematics may have been rare or even non-existent. Recall in this context that Buddhists did not practise mathematics, and left this task essentially to Brahmins.[6] Mathematicians, as a result, did not feel threatened, and they did not have to prove that their theorems were correct. The result of this situation we have seen: some of Bhāskara's theorems were in actual fact incorrect, but no one over a period of one and a half centuries (or more) noticed.

There is one important exception to what I have just said. In one passage Bhāskara discusses an opinion with which he disagrees. A central point of disagreement is the value of π. Bhāskara's opponent holds that it is √10. Bhāskara disagrees. In his comments on *Gaṇitapāda* verse 10 we read, for example:[7]

> [Question:] Isn't there the following saying: the circumference of a circle is the square root of 10 times the diameter?
> [Response:] Here too, [the claim] that the circumference of a diameter that equals one is the square root of 10 is mere tradition, and no demonstration (*upapatti*).

The line that Bhāskara here quotes — "the circumference of a circle is the square root of 10 times the diameter" — is in Prakrit. This suggests that he quotes from a Jaina text. This impression is strengthened by the fact that the quoted line assigns the value √10 to π. This value, as Kim Plofker points out in her book *Mathematics in India* (2009: 59) "is so routinely used in Jain texts that it is often known as the 'Jain value' for π." Indeed, it is still employed in the *Gaṇitasārakaumudī* of Ṭhakkura Pherū in first quarter of the 14th century, but also in earlier texts such as the *Tiloyapaṇṇattī* (Skt. *Trilokaprajñapti*), a Prakrit Digambara work on cosmology,[8] and in Pādalipta's *Joïsakaraṇḍaga* (Skt. *Jyotiṣkaraṇḍaka*).[9] Mahāvīra's

[6] See my book *Buddhism in the Shadow of Brahmanism* (Bronkhorst 2011).

[7] Āryabhaṭīya Bhāṣya p. 72 l. 11-15: *nanu cāyam asti: vikkhaṃbhavaggadasaguṇakaraṇī vaṭṭassa parirao hodi (viṣkambhavargadaśaguṇakaraṇī vṛttasya pariṇāho bhavati) iti / atrāpi kevala evāgamaḥ naivopapattiḥ /.* Cf. Keller 2006 I: 52; II: 48-49.

[8] SaKHYa 2009: xxvii, 144-145. Not only Jainas used this value for π: also Brahmagupta (a contemporary of Bhāskara I) does so (Sarasvati Amma 1979: 62), as does Śrīdhara (eighth or ninth century) in his *Triśatikā* (SaKHYa 2009: xxvii, 144). Also the *Pauliśa Siddhānta* (referred to by Varāha Mihira, 6th century) appears to

Gaṇitasārasaṅgraha gives both 3 and √10 as values for π.[10] The value √10 occurs perhaps for the first time in a Jaina canonical text, the *Sūr[iy]apaṇṇattī* (Skt. *Sūryaprajñapti*).[11] Bhāskara here criticizes this value by stating that it merely gives expression to a tradition (*āgama*), without demonstration. Here he uses the expression *upapatti*, which some translate 'proof', but for which 'demonstration' seems more appropriate.

It is hard to figure out what 'demonstration' would convince Bhāskara of the truth of the claim that $\pi = \sqrt{10}$.[12] As a matter of fact, he goes on criticizing the claim that $\pi = \sqrt{10}$ at great length. One way in which he tries to demonstrate the insufficiency of this value for π is by showing that it leads to totally unacceptable consequences. He does so with the help of a concrete example: a circle whose diameter (CE) is 52, and the length of the sagitta (CD) is 2. A theorem allows Bhāskara to calculate the chord (AB). In fact, the theorem says that AD = 10, so that AB = 20.

Interestingly, Bhāskara cites this theorem in Prakrit. This suggests that he found it, like the line in Prakrit we considered earlier, in a Jaina work on geometry, presumably the same one where he found the earlier line. However, he did not need to quote this theorem from a Jaina work: it is also one of his own theorems, formulated differently, and in Sanskrit, by Āryabhaṭa in *Gaṇitapāda* 17cd. This allows us to conclude that Bhāskara is building up an argument based on the words of his opponent.

As a next step in this argument, Bhāskara now cites a line in Sanskrit that he calls a *sūtra*. Given the context, we must assume that this *sūtra*, too, comes from a Jaina text, perhaps the same as before, in which case we must assume that it was a text partly in Prakrit and partly in Sanskrit. Let us look at what the *sūtra* says.

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have used this value; Kaye 1915: 10. Hayashi (1997: 194) adds *Paitāmahasiddhānta* (ca. 425 CE), Varāhamihira (ca. 550 CE), Āryabhaṭa II (ca. 950 CE) and the *Sūryasiddhānta* (ca. 800 CE) to this list of non-Jaina texts and authors.

9 Sarasvati Amma 1979: 63. See also Jain 1977: 19 (Mahāvīra); Datta 1929: 124 (Umāsvāti); Hayashi 1997: 194 (*Jambuddīvapaṇṇattī*); Hayashi estimates that the *Sūriyapaṇṇattī* and the *Jambuddīvapaṇṇattī* "seem to have taken their present forms by the fifth century AD, when the last Jaina 'council' (or recitation) was held at Valabhī."

10 Raṅgācārya 1912: 189 (7.19); 200 (7.60).

11 *Sūryaprajñapti* I.7 and IV according to Sarasvati Amma 1979: 62 n. 1. See further Datta 1929: 131. Still in 1816, Taylor 1816: 2 speaks of "Jaina priests, many of whom profess astrology."

12 For modern speculations, see Sarasvati Amma 1979: 65, which refers to Moritz Cantor's 1907: 647-649 *Vorlesungen über Geschichte der Mathematik*.

The *sūtra* gives indications as to how to calculate the length of an arc:[13]

> "The sum of a quarter of the chord and half the sagitta, multiplied by itself, ten times that, the square root of that."

In the example under consideration, the quarter of the chord is 5. Half the sagitta is 1. The sum of these two is 6. 6 x 6 = 36. 10 x 36 = 360. The square root of 360 is supposed to be the length of the arc AB.

However, the length of the chord AB is 20, which is the square root of 400. Clearly the square root of 360 is less than the square root of 400. But the arc AB cannot be shorter than the chord AB. In other word, this outcome is impossible.

What went wrong? At first sight, Bhāskara blames the value assigned to π. As he puts it:[14] "The calculation of an arc on the basis of the assumption that the measurement of a circumference is made with the square root of ten, is not always [possible]." This, however, makes no sense. Because the approximate value of π which he, following Āryabhaṭa's *Gaṇitapāda* verse 10, does accept (π = 3.1416),[15] if used with the same but adjusted formula, leads to the same absurdity (6 x 3.1416 = 18.8496 for the length of arc; 20 for the chord). In other words, not only the Jainas' value of π is wrong, but so is the formula they use to calculate the length of the arc. Bhāskara's criticism does not just concern the value of π, but those who accept that value of π, i.e., the Jainas who accept that $\pi = \sqrt{10}$.

What does all this teach us? It shows that the same Bhāskara who could not discover mistakes made in his own tradition, has no difficulty finding mistakes in a tradition different from his own. And this tradition different from his own, in this case, appears to be the, or a, Jaina tradition of geometry.

However, it looks as if Bhāskara's criticism of the Jaina tradition contains a logical error. The *sūtra* he criticizes is no doubt incorrect, but this fact has nothing to do with the value that the Jainas attribute to π. Whatever value one attributes to π, the *sūtra* will always lead to an absurdity. It is unlikely that a contemporary philosopher trained in logic would have made such a mistake.

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[13] *Āryabhaṭīya Bhāṣya* p. 74 l. 10-11: *pṛṣṭhānayane sūtram āryārdham:* ***jyāpādaśarārdhayutiḥ svaguṇā [daśasaṅguṇā karaṇyas tāḥ].*** Cf. Keller 2006 I: 54-55; II: 48.

[14] *Āryabhaṭīya Bhāṣya* p. 74 l. 19: *pṛṣṭhānayanam api ca daśakaraṇīparidhiprakriyāparikalpanayā sadā na [bhavati].*

[15] "[I]t should be recalled that this is essentially the value Ptolemy had used" (Merzbach & Boyer 2011: 190). It is also the value proposed by Bhāskara II in his *Līlāvatī* (Taylor 1816: 94).

Having made a comparison with the history of Indian philosophy, it is almost a pity that there were apparently so few occasions for confrontation between Bhāskara's and other traditions of geometry in classical India. If there had been, each of these traditions might have developed a deeper critical sense, first with regard to others, but ultimately also with regard to themselves. The outcome might have been positive for all. For one thing, more self-criticism might have induced Bhāskara and no doubt others to critically evaluate the theorems that had been handed down to them and ask themselves why they should accept them. Incorrect theorems might in this way have been weeded out. In reality no such thing happened — not at least at the time of Bhāskara.

We do not know whether mathematicians in the Jaina tradition cared about Bhāskara's criticism. We have seen that they continued to assign the value $\sqrt{10}$ to π, until many centuries after Bhāskara. The problem that Bhāskara found with this value, as we have seen, was linked to a rule about the length of the arc of a circle. Interestingly, many Jaina mathematicians used an altogether different rule to calculate the length of an arc. If **a** is the length of the chord, **h** that of the sagitta, and **b** the length of the arc, many Jaina mathematicians used the following rule:

$$a^2 + 6h^2 = b^2$$

This rule is obviously not open to Bhāskara's criticism,[16] for clearly here the arc (**b**) is longer than the chord (**a**). This better rule was already used by Umāsvāti (well before Bhāskara),[17] and later by the mathematicians Mahāvīra (9th century CE), Āryabhaṭa II (around 1000 CE), Śrīpati (11th century), and Ṭhakkura Pherū (14^{th} century) (not all of them Jainas).[18] Interestingly, it has been suggested that this formula is "obtained from the approximate relationship, $a^2 + (\pi^2 - 4)h^2 = b^2$ with $\pi = \sqrt{10}$."[19] If so, the value $\pi = \sqrt{10}$ remained popular among Jaina and non-Jaina mathematicians alike, but the formula used to calculate the length of the arc was different from the one criticized by Bhāskara.

However this may be, it seems clear that geometry at the time of Bhāskara I was practised in different schools. To some extent these schools were independent of each other and concentrated on their own traditional teachings. If these traditional teachings were incorrect, they were preserved without anyone being aware of this fact. Criticism only came

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[16] The Jaina rule that Bhāskara criticizes can be represented as follows: $5a^2 + 20h^2 + 20ah = 8b^2$.

[17] Datta 1929: 124.

[18] SaKHYa 2009: xxii, 145-146.

[19] SaKHYa 2009: 146.

into play when representatives of different schools confronted each other. Only in such cases was there place for criticism. Unfortunately such confrontations were, at the time of Bhāskara, too infrequent to allow a general atmosphere of criticism to become part of the traditions.

APPENDIX: SAGITTAS AND CHORDS

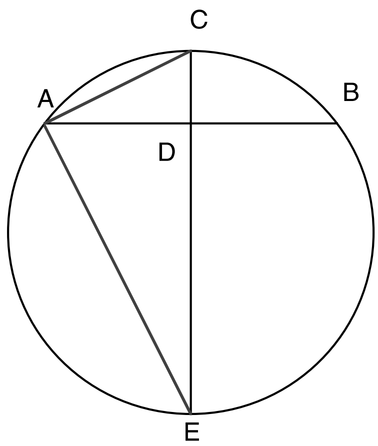

Fig. 1: CD x DE = AD^2

Gaṇitapāda 17cd: *vṛtte śarasaṃvargo 'rdhajyāvargaḥ sa khalu dhanuṣoḥ*
"In a circle the product of the two sagittas of the two arcs [that together constitute the circle] equals the square of half the chord."

That is to say: CD x DE = AD^2.

p. 73 l. 2-4:
ogāhūṇam vikkhambham egāheṇa saṃguṇam kuryāt |
cauguṇiassa tu mūḷam jīvā savvakhattāṇam ||
[avagāhonam viṣkambham avagāheṇa saṅguṇam kuryāt |
caturguṇitasya tu mūlam sā jīvā sarvakṣetrāṇām || *]*
Tr. Keller 2006: 53:
"The diameter decreased by the penetration (*avagāha*) should be multiplied by the penetration / Then the root of the product multiplied by four is the chord of all fields."

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